Appendix B – FAO-56 (Smith, 1991) Guidelines for Estimating Grass-Reference Evapotranspiration Based on the Penman-Monteith Method

$$\lambda ET_o = \frac{\Delta(R_n - G) + \rho c_p (e_s - e_a)1/r_a}{\Delta + \gamma(1 + r_c / r_a)}$$

λET_o	:	latent heat flux of evaporation for a standard crop	[kJ m^{-2} s^{-1}]
Δ	:	slope saturated vapor pressure-temperature curve at mean air temperature	[kPa °C^{-1}]
γ	:	psychrometric constant	[kPa °C^{-1}]
R_n	:	net radiation	[kJ m^{-2} s^{-1}]
G	:	soil heat flux	[kJ m^{-2} s^{-1}]
c_p	:	specific heat of moist air	[kJ kg^{-1} °C]
ρ	:	atmospheric density	[kg m^{-3}]
c_p	:	specific heat of moist air	[kJ kg^{-1} °C^{-1}]
e_s	:	saturation vapor pressure at mean air temperature	[kPa]
e_a	:	actual vapor pressure or saturation vapor pressure at dew point temperature (T_{dew})	[kPa]
r_c	:	crop canopy (bulk stomata) resistance	[s m^{-1}]
r_a	:	aerodynamic resistance	[s m^{-1}]

Parameters used in equation above are estimated as follows:

Latent heat of vaporization (λ):

$$\lambda = 2.501 - (2.361\text{E} - 3)T$$

λ	:	latent heat of evaporation	[MJ kg^{-1}]
T	:	daily mean air temperature	[°C]

Psychrometric Constant (γ):

$$\gamma = 0.00163\frac{P}{\lambda}$$

P	:	atmospheric pressure	[kPa]

Specific heat of moist air (c_p):

$$c_p = \gamma \frac{0.622\lambda}{P}$$

Saturation vapor pressure (e_s):

$$e_s = 0.5(0.611\exp(\frac{17.27T_{\max}}{T_{\max}+237.3}) + 0.611\exp(\frac{17.27T_{\min}}{T_{\min}+237.3}))$$

Actual vapor pressure (e_a):

$$e_a = 0.611\exp(\frac{17.27T_{dew}}{T_{dew}+237.3})$$

Or

$$e_a = \frac{RH_{\max}/100 * e_s(T_{\min}) + RH_{\min}100 * e_s(T_{\max})}{2}$$

T_{dew}:	point temperature	[°C]
RH_{max}:	daily maximum relative humidity	[%]
RH_{min}:	daily minimum relative humidity	[%]
T_{max}:	daily maximum air temperature	[°C]
T_{min}:	daily minimum air temperature	[°C]

where

$$e_s(T) = 0.611\exp\left(\frac{17.27T}{T+237.3}\right)$$

Atmospheric density (ρ):

$$\rho = \frac{1000P}{T_{kv}R} = 3.486\frac{P}{T_{kv}}$$

R :	Specific gas constant = 287	[J kg^{-1} K^{-1}]
T_{kv} :	Virtual temperature	[K]

Virtual temperature (T_{kv}):

$$T_{kv} = T_k(1 - 0.378\frac{e_a}{P})^{-1}$$

T_k :	daily mean air temperature	[K]

Slope vapor pressure curve (Δ):

$$\Delta = \frac{4098e_s}{(T+237.3)^2}$$

T :	daily mean air temperature	[°C]

Soil heat flux (G):

$$G = c_s d_s \left(\frac{T_n - T_{n-1}}{\Delta t}\right)$$

G	:	soil heat flux	[MJ m^{-2} d^{-1}]
T_n	:	mean temperature on day (or month) n	[°C]
T_{n-1}	:	mean temperature on previous day (or month) n-1	[°C]
Δt	:	length period	[d]
c_s	:	soil heat capacity = 2.1	[MJ m^{-3} °C^{-1}]
d_s	:	estimated effective soil depth = 0.18 for daily temperature fluctuations (Wright and Jensen, 1972)	[m]

Note: The soil heat flux can generally be neglected for daily timesteps.

Aerodynamic resistance (r_a):

$$r_a = \frac{\ln\left(\frac{z_m - d}{z_{om}}\right) . \ln\left(\frac{z_h - d}{z_{oh}}\right)}{k^2 U_z}$$

r_a	:	aerodynamic resistance	[s m^{-1}]
z_m	:	height of wind speed measurements	[m]
z_h	:	height of temperature and humidity measurements	[m]
k	:	von Karman Constant = 0.41	
U_z	:	wind speed at height z_m	[m s^{-1}]
d	:	zero plane displacement of wind profile = 0.08	[m]
z_{om}	:	roughness parameter for momentum = 0.015	[m]
z_{oh}	:	roughness parameter for heat and water vapor = 0.0015	[m]

Crop canopy resistance (r_c):

$$r_c = \frac{R_l}{0.5 LAI} = \frac{200}{LAI}$$

R_l	:	average 24-hour stomata resistance of single leaf = 100	[s m^{-1}]
LAI	:	leaf area index = 24 h_c for clipped grass	

For reference crop, h_c = 0.12, hence LAI = 2.88 and r_c = 70 [s m^{-1}].

Relative distance from the sun to the earth (d_r):

$$d_r = 1 + 0.033 \cos\left(\frac{2\pi J}{365}\right)$$

J : Julian day of the year

Declination of the sun (δ):

$$\delta = 0.409\sin(\frac{2\pi J}{365} - 1.39)$$

Sunset hour angle (ω_s):

$$w_s = \arccos(-\tan\varphi \tan\delta)$$

δ	:	declination of sun	[rad]
φ	:	station latitude	[rad]

Extraterrestrial solar radiation (R_a):

Extraterrestrial radiation can be calculated from latitude and time of year by integrating the instantaneous radiation intensity at the outer atmosphere from sunrise to sunset in

$$R_a = \frac{24*60}{\pi} G_{sc} d_r (\omega_s \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\omega_s)$$

Ra	:	extraterrestrial solar radiation	[MJ m^{-2} d^{-1}]
G_{sc}	:	solar constant = 0.8202 (Duffie and Beckman, 1991)	[MJ m^{-2} min^{-1}]

Net shortwave radiation (R_{ns}):

Net shortwave radiation is the solar radiation received by the surface taking into account losses due to reflection.

$$R_{ns} = (1-\alpha)R_s$$

R_{ns}	:	net shortwave solar radiation (shortwave)	[MJ m^{-2} d^{-1}]
α	:	albedo or canopy reflection coefficient = 0.23 overall average for grass	
R_s	:	incoming solar radiation (shortwave)	[MJ m^{-2} d^{-1}]

Clear-sky shortwave radiation (R_{so}):

$$R_{so} = \tau_o R_a = (a_s + b_s + 2*10^{-5} * z)R_a$$

R_{ns}	:	cloudless shortwave solar radiation (shortwave)	[MJ m^{-2} d^{-1}]
τ_o	:	clear-sky transmissivity = a_s + b_s + $2*10^{-5}z$	

The above equation is for cases when calibrated values of the Angstrom values are not available, therefore, Angstrom values of a_s = 0.25 and b_s = 0.50 are used.

Cloudiness adjustment factor (f_{cl}):

$$f_{cl} = a_c \frac{R_s}{R_{so}} + b_c$$

where a_c = 1.35 and b_c =0.35 as recommended by FAO (1977).

Net Emissivity (e'):

$$e' = 0.34 - 0.14\sqrt{e_d}$$

Net long-wave radiation (R_{nl}):

Net longwave radiation is the difference between thermal radiation from vegetation and soil to the atmosphere and reflected radiation from the atmosphere and clouds, and can be estimated using:

$$R_{nl} = f_{cl}(e')\sigma \frac{T_{max}^4 + T_{min}^4}{2}$$

R_{nl} :	net long-wave radiation	[MJ m^{-2} d^{-1}]
T_{max}:	daily maximum air temperature	[K]
T_{min}:	daily minimum air temperature	[K]
σ :	Stefan-Boltzmann constant	[4.903 10^{-9} MJ K^{-4} m^{-2} day^{-1}]

Net radiation (R_n):

Net absorbed radiation is the difference between absorbed incoming shortwave solar radiation and net outgoing long-wave radiation.

$$R_n = R_{ns} - R_{nl}$$

R_{nl} :	net radiation	[MJ m^{-2} d^{-1}]