MSE Assessor Documentation
& SFRSM Regional Coordination LP Problem Formulation

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Abstract

This document details the implementation of the MSE Assessor, which includes the use of MSE-HSE iterations. A secondary objective is to formulate an expression for regional coordination of SFRSM water resources based on a mathematical programming language (MathProg) applied to a linear programming solver.

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1 Introduction

The South Florida Regional Simulation Model (SFRSM) is a long term project to implement the Regional Simulation Model (RSM) [SFWMD1 2005] over a majority of the geographical area under jurisdiction of the South Florida Water Management District (SFWMD). While the Hydrologic Simulation Engine (HSE) component of the RSM is well developed and peer-reviewed, the Management Simulation Engine (MSE) component is currently under development. The underlying architecture of the MSE is well established [SFWMD2 2005], and has been successfully applied in subregional model applications. However, attempts to simulate the complex, interdependent operational control scenarios required for the SFRSM with the previously existing MSE tools proved problematic. One of the primary difficulties was the imposition of a daily timestep. The large simulation interval expands inaccuracies in the estimation of hydrological state variables within the MSE. This issue is discussed in sections 5. Another issue was that expression of operational policies for coupled structures, and those which rely on multiple, remote data sources would have resulted in an implementation that was deemed overly complex and potentially inextensible.

Recognition that these MSE implementation issues would be detrimental to the long term evolution and support of the SFRSM motivated the development of two new abstractions within the RSM: MSE Network, and Assessors. The MSE Network facilitates mapping of hydrological objects within the HSE to managerial objects such as water control units[SFWMD2 2005]. Assessors were conceived as specialized data processing modules able to assess, filter and transform individual data into a cohesive, synoptic assessment of hydrological states used as decision variables. Recent development has resulted in two implementations of Assessors: the WCU Assessor, and the MSE Assessor. As a matter of convenience, these implementations are hybrid Assessor/Supervisor constructs, they directly control water flows at simulated hydraulic structures.

The primary issue to be addressed is:

1. Estimate interbasin management flows which are compatible with the hydrological model state response over periods of 1 day, and which satisfy operational constraints & objectives

On a subregional scale, this problem has been addressed by both the WCU

Assessor, and the MSE Assessor. However, there is a motivation to model this problem with a LP or NL solver in an attempt to reduce the number of state-estimation iterations currently employed in the MSE Assessor management algorithm. This is discussed in section 7.

On a regional scale, the problem is congruent with many historical applications of optimization techniques aimed at estimating water resource allocations in a multibasin network flow problem. In the context of the SFRSM this would include special operations for regional water supply or CERP projects. Additionally, capabilities are needed to compliment the current assessor implementations with regional scale management functions. This is discussed in section 8.

1.1 † Contributors

The work described here is based on a lengthy collaboration among the following: Randy Van Zee, Raul Novoa, Michelle Irizarry, Fawen Zheng, Ray Santee, Dave Welter, Wasantha Lal.

2 Operational Problem Statement

Consider a collection of managed basins with controlled flow conduits between basins. For example, figure 1 represents a water control network with basins B1 through B6 where q_{12} indicates the flow from basin B1 to basin B2.

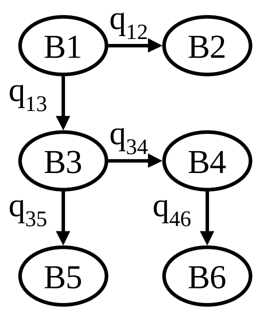

Figure 1: Water control network of connected basins.

In the context of the RSM, a basin consists of a set of aquifer/land surface mesh cells, and a collection of canal segments or other waterbodies such as lakes, within the confines of the basin cells. To facilitate representation of a basin canal network as a single, managed water resource, the abstraction Water Control Unit (WCU) refers to the collection of canal segments within the basin. Each WCU has associated with it a set of operational constraints. For example a maintenance level specifies a water level minimum target value for water supply or environmental purposes, while a flood control level indicates a WCU maximum water level target value.

The flow conduits between basins represent controlled hydraulic structures. The q's are actually flows between WCU canal segments through a structure. Each structure may have operational constraints such as a maximum flow capacity, or maximum/minimum flow values dictated by water supply or environmental objectives. Functionally, the flow between two basins depends on both hydrological state information (s), and a managerial control value (χ). The control value is itself a function of s, as well as a function of operational constraints and objectives (λ). This can be expressed as:

$$q = f(s, \chi(s, \lambda)) \tag{1}$$

As described in a later section, the state information constitutes observations of a nonlinear dynamical system.

The flows in figure 1 are instantaneous values which vary continuously, and which we assume are differentiable as many times as needed. In the context of an RSM implementation used as a water resource management evaluation or planning tool, the flow metric of interest is typically a cumulative flow over a period of time which meets the management objectives. Accordingly, we define the cumulative flow from basin n to basin m over the time period starting at $\mathrm{t_s}$ and ending at $\mathrm{t_e}$ as:

$$\mathrm{Q_{nm}^{se}} = \int_{\mathrm{t_s}}^{\mathrm{t_e}} \mathrm{q_{nm}(t)dt} \tag{2}$$

Specifically, in the RSM application to the South Florida region, a daily timestep has been specified as the simulation increment, i.e. $\Delta\mathrm{t} = \mathrm{t_e} - \mathrm{t_s} =$ 1 day. This constraint is consistent with a large body of existing simulation results and database of historical structure flows and water levels. Further, the model period of record can span 30 years or more, and the large timestep is desirable to limit simulation run times. We will denote the cumulative interbasin flow over a daily time period as $\mathrm{Q_{nm}}$. Estimation of the cumulative flows Q_{nm} over a simulation timestep of 1 day is the primary objective of the MSE Assessor (section 4), and of the linear programming models which are under development (sections 7 and 8).

3 RSM State Estimation

RSM is a state estimator. We denote estimates of variables with italics. RSM allows independent abstraction of hydrological and managerial state variables through the interoperation of the Hydrologic Simulation Engine (HSE) and the Management Simulation Engine (MSE). HSE provides hydrological state evaluations, s, while MSE facilitates estimation of controlled variables such as q_{nm}. RSM also provides for transformation of s with a set of data filters known as Assessor's. A schematic of the overall RSM state information cyclic flow is shown in figure 2.

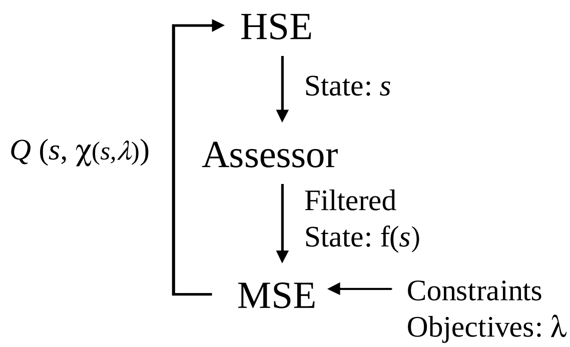

Figure 2: RSM schematic.

3.1 Linear Model

HSE facilitates estimation of the hydrological states through a linearized diffusion flow formulation. Typically, a linear model is represented as a superposition of weighted states and forcing (or basis) functions:

$$s(\mathrm{n}) = \sum_{\mathrm{i}=1}^{\mathrm{N}} a_{\mathrm{i}} s(\mathrm{n}-\mathrm{i}) + \sum_{\mathrm{j}=1}^{\mathrm{M}} b_{\mathrm{j}} \Phi(\mathrm{n}-\mathrm{j}) \tag{3}$$

where the a_{i} and b_{j} are model coefficients and Φ represents the forcing terms. The task is then one of judiciously selecting the coefficients to conform the model results with the observations. In HSE, the hydrological representation is [SFWMD1 2005]

$$\boldsymbol{A(H)} \cdot \frac{d\boldsymbol{H}}{dt} = \boldsymbol{q(H)} + \boldsymbol{S(H)} \tag{4}$$

where $\boldsymbol{H}$ is a vector of finite volume waterbody states, $\boldsymbol{q(H)}$ is a vector containing the summation of flow entering the waterbodies, $\boldsymbol{S(H)}$ are non-gradient driven fluxes (source terms) and $\boldsymbol{A(H)}$ is a diagonal matrix whose elements contain the effective areas of the waterbodies. The flows $\boldsymbol{q(H)}$ are linearized through use of a global flow resistance matrix $\boldsymbol{M(H)}$

$$\boldsymbol{q(H)} = \boldsymbol{M(H)} \cdot \boldsymbol{H} \tag{5}$$

The flows of equation 5 are solved with a PETSC sparse linear system solver [SFWMD1 2005]. Once a solution of the hydrological states are available, dynamical evolution of the simulation is specified as

$$s(\mathrm{n}+1) = s(\mathrm{n}) + \boldsymbol{A_{\mathbf{n}}} \cdot \Delta H \tag{6}$$

which is a special case of equation 3.

While many of the linearizations are well characterized, it is possible that state variable regimes which invalidate the linear assumptions could precipitate unanticipated model behaviors. The chosen spatiotemporal discretizations of the model representation are also capable of introducing nonlinear simulation artifacts. In addition, there are likely nonlinear system variables which are ignored by the model equations. Further, there may be inherent limitations in the exclusion of hydrodynamic momentum terms from the model formulation wherein stream flow dynamics are approximated. It is known that the hydrological states are expressions of the dynamic evolution of a nonlinear, chaotic timeseries [Park 2005], however, the significant dissipation inherent in the physical system allows reasonable approximations given well behaved linearizations ($ds/d\mathrm{t} \propto \mathrm{t}$) and small enough simulation timesteps ($\Delta\mathrm{t} \to 0$).

3.2 Nonlinear Model

In order to model a dynamical system it is assumed that a set of differential equations or discrete time evolution rules govern the behavior of the f system variables contained in the vector $\boldsymbol{\mu}(\mathrm{t})$

$$\frac{d\boldsymbol{\mu}(\mathrm{t})}{d\mathrm{t}} = \mathbf{G}(\boldsymbol{\mu}(\mathrm{t})) \tag{7}$$

where $\mathbf{G}$ is a vector field that is continuous in its variables and which is differentiable as needed. The value of f defines the number of independent equations required to form an orthogonal basis to describe the system dynamics, and corresponds to the number of phase-space dimensions required to completely unfold the attractor so that false projections and crossings of $\boldsymbol{\mu}$ are eliminated.

In most modeling applications one has access to sampled versions of the process dynamics at a fixed spatial point

$$s(\mathrm{n}) = s(\mathrm{t}_0 + \mathrm{n}\tau_s) \tag{8}$$

where t_0 is the initial time and τ_s the sampling interval for the n^{th} observation. The discrete time extension of equation 7 is specified by a map from vectors in $\Re^f$ to other vectors in $\Re^f$, each at a discrete time $\boldsymbol{\mu}(\mathrm{n}) = \boldsymbol{\mu}(\mathrm{t}_0 + \mathrm{n}\tau_s)$

$$\boldsymbol{\mu}(\mathrm{n}+1) = \mathbf{F}(\boldsymbol{\mu}(\mathrm{n})) \tag{9}$$

This expression defines the evolution equation of the dynamical system, the vector field $\mathbf{F}$ encapsulates parameters which reflect the physical properties of the system as well as external influences of forces and boundary conditions. From a modeling perspective, the approach is to identify parameterized nonlinear functions $\mathbf{F}(\boldsymbol{\Psi})$ which map $\boldsymbol{\mu}(\mathrm{n})$ into $\boldsymbol{\mu}(\mathrm{n}+1) = \mathbf{F}(\boldsymbol{\mu}(\mathrm{n}), \boldsymbol{\Psi})$ where one has applied appropriate fit criteria to evaluate the parameters $\boldsymbol{\Psi}$.

The idea is to let the data itself dictate the essential invariant features of the process dynamics. Once the phase-space invariants are identified, interpolation and projection of the process trajectories can provide nonlinear estimators and predictors. The modeler is then faced with the task of coupling physical significance to the revealed invariants.

In terms of HSE as a state estimator of nonlinear dynamical hydrological processes, the linearized finite volume solution of equation 6 is an approximation of a general nonlinear evolution as described by equation 9. Based on the realization that the hydrologic states are chaotic, it is assumed that the flows described by equation 1 are also expressions of a nonlinear dynamical process. One of the challenges posed is to identify whether a linear formulation for controlled interbasin flows can robustly approximate the nonlinear dynamics.

4 MSE Assessor

The MSE Assessor refers to a combined state variable assessor and inter-basin water flow supervisor. It is based on methodology implemented in the South Florida Water Management Model (SFWMM) [SFWMD4 1999], and on code from the WCU Assessors developed in collaboration with Randy Van Zee. The MSE Assessor estimates controlled basin structural outflows to satisfy both water supply and flood control operational constraints.

The MSE Assessor is designed to estimate the cumulative flows Q_{nm} over a simulation timestep of 1 day. Currently, this is done through an iterative procedure which updates state information from the HSE for refined flow estimates in the MSE Assessor. The iterative scheme is detailed in section 6. One of the objectives of this research problem statement is to evaluate the feasibility of reducing the number of HSE state iterations through the use of an optimization algorithm. This issue is explored in section 7. The remainder of this section serves as documentation of the currently implemented MSE Assessor for the SFRSM project.

4.1 Assess Function

The MSE Assessor interface in the RSM is contained in the `Assess()` function. A flowchart schematic representation of this function is shown in figure 3.

The `Assess()` function is executed before each solution of the HSE state equations. Once the WCU outlet flows (Q_{nm}) are estimated, these flows are imposed as boundary conditions on the HSE solution. The `Assess()` function has three primary operations:

1. Assess the volumetric supply or demand (needs) of each WCU
2. Accumulate the WCU needs across multiple, interconnected WCU's
3. Route WCU outlet flows to satisfy water supply needs and flood control objectives

The water supply needs are evaluated by the three functions `WSNeeds()`, `ReserveNeeds()` and `LocalExcess()`. These three functions perform essentially the same computation: estimate the volume of water needed to raise or lower a WCU to a target level, but with respect to three different target water levels: `MaintLevel`, `ResLevel` and `LocalLevel` respectively. The function which computes the water supply needs for each target is the `TargetVolume()` function, which is detailed in section 4.2.

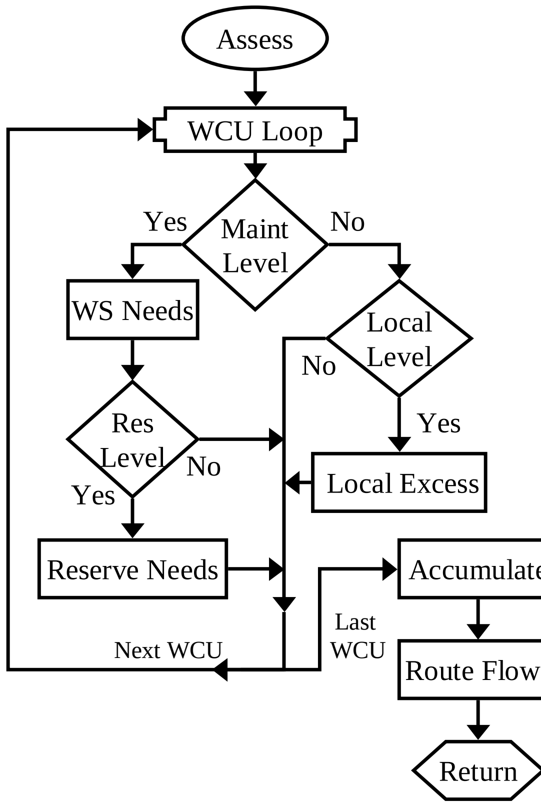

Figure 3: Flowchart of MSE Assessor main function: Assess()

`WSNeeds()` is the volume of water required to bring the downstream end of a WCU to it's maintenance level. `ReserveNeeds()` is a smaller volume than `WSNeeds()` and is the volume that will be supplied to a WCU if water availability from upstream sources is limited. `LocalExcess()` is the volume that a flow-thru WCU can provide as local water supply to downstream WCUs. If a flow-thru WCU has excess volume, `LocalExcess()` will return a negative volume.

4.2 TargetVolume() function

The primary computation of the `TargetVolume()` function is an estimate of the total volumetric water differential needed to raise or lower the water level of a WCU to satisfy a target water level. This computation is specific to water supply needs or excesses, flood control releases are computed separately in the `RouteFlow()` function as described in section 4.4.

Figure 4 indicates a cross-sectional view of a WCU consisting of 4 HSE canal segments. The water level difference between the initial level and the target level at the downstream control point is denoted ΔH_T. Once this target differential is computed, it is added as an offset to each canal segment water level in the WCU. These 'adjusted' water levels constitute a WCU water level profile which defines the target water levels over the entire WCU.

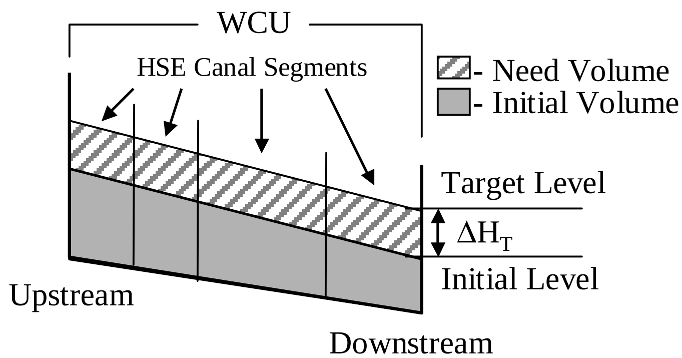

Figure 4: WCU volume and target level for water supply needs.

In the first HSE iteration, it is assumed that the target profile is parallel to the previous time step profile. However, in subsequent iterations, the target profile is assumed parallel to the profile obtained in the previous sub-timestep iteration. Section 6 discusses the MSE - HSE iterations.

Once the adjusted target profile is available, each canal segment in the WCU is processed to estimate:

1. Volume required to raise/lower the initial level to the target: V_{H_T}
2. Canal segment water stage: $s_s(\mathrm{n}) = s_s(\mathrm{n}-1) + \alpha_h \Delta H$
3. Aquifer cell water stage: $s_c(\mathrm{n}) = s_c(\mathrm{n}-1) + \alpha_h \Delta H$
4. Volume of canal/aquifer seepage: $V_{SP} = f(s_s(\mathrm{n}), s_c(\mathrm{n}))$
5. Volume canal overbank flow: $V_{OB} = f(s_s(\mathrm{n}), s_c(\mathrm{n}))$
6. Volume of levee seepage: $V_{LV} = f(s_s(\mathrm{n}), s_c(\mathrm{n}))$
7. Volume of boundary condition flows: V_{BC}
8. Volume of WCU unmanaged (passive) structure inlets: $V_I(s_s(\mathrm{n})$
9. Volume of WCU unmanaged (passive) structure outlets: $V_O(s_s(\mathrm{n})$

where s_s is the canal segment water level, s_c the aquifer cell water level, α_h an implicit/explicit numerical solution weight [SFWMD1 2005], and ΔH the previous HSE solution of state change. Boundary condition flows include HSE watermovers defined by HSE boundary conditions, for example, a canal segment may have a water stage boundary condition, or flow boundary condition defined by a timeseries [SFWMD1 2005]. These estimates are then accumulated into a final value of volumetric water supply need (WSN) for the WCU:

$$V_{WSN} = \sum_{i=1}^{\mathrm{N}} \{V_{H_T i} + V_{SPi} + V_{OBi} + V_{LVi} + V_{BCi} + V_{Ii} + V_{Oi}\} \qquad (10)$$

where N is the total number of canal segments in the WCU. The value of V_{WSN} is contained within the domain of $\Re$. Positive V_{WSN} indicates the deficit volume which needs to be added to the WCU to meet the target level, negative V_{WSN} signifies a volume of excess water above the target level. The estimates of V_{WSN} are stored in data objects of each respective WCU for subsequent reference.

4.3 Accumulate() function

After each WCU has been evaluated for water supply needs, the `Accumulate()` function processes the entire WCU network to estimate the cumulative water supply need at each WCU inlet. Figure 5 depicts a schematic flowchart of the `Accumulate()` function.

The `Accumulate()` function contains three internal loops:

1. Process all WCU's from downstream to upstream (index(i))
2. Process all outlets of a WCU to accumulate the CWS (index(j))
3. Process all inlets of a WCU to compute capacity weight β (index(k))

The first (outer) loop ensures that all WCU's in the flow control network are processed. The order of processing is determined by the structure of the MSE Network definition file [SFWMD3 2005]. The first step in this loop is to initialize the cumulative water supply need (CWS) for each WCU as either the water supply need (WSN, positive or negative) or local excess (LEX, negative) volume which were previously computed for each WCU according to equation 10 (see figure 3).

The second loop then accesses all water supply outlet structures of the current WCU, and accumulates their downstream CWS with the current

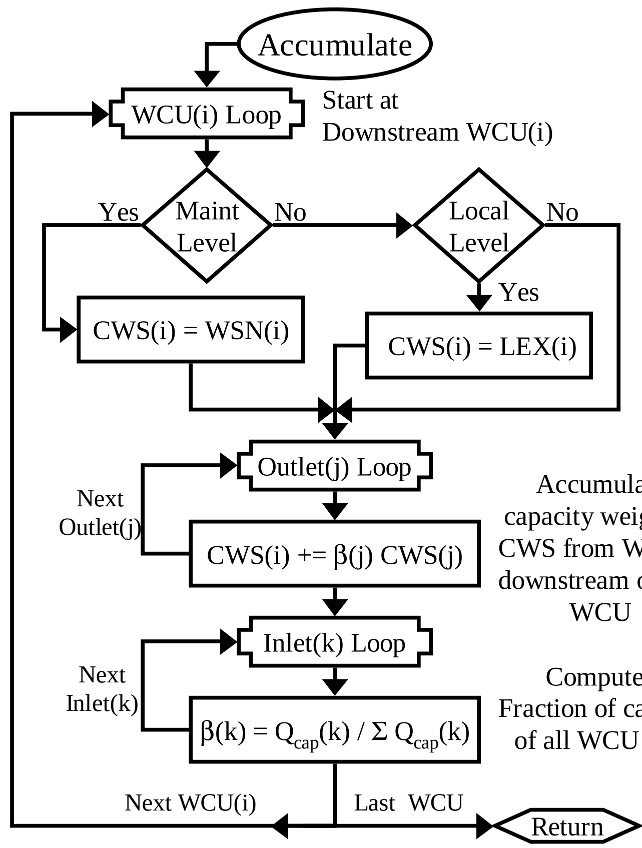

Figure 5: Flowchart of MSE Assessor function: Accumulate()

WCU. This accumulation is weighted by the flow capacity of the downstream outlet structures. Note that CWS_j is limited to be greater than or equal to 0 since excess water cannot be transferred upstream. The third loop computes the capacity weight β for all water supply inlets of the current WCU. Since the processing of WCU's is from downstream to upstream, a value of β is always available for WCU outlets in the second loop. β is used for WCU's with multiple water supply inlets to assign fractions of CWS to be met through different routes. The inherent assumption is that routes with more capacity will be used proportionaly more for water supply.

4.4 RouteFlow() function

The functions described previously all fall under the functional classification of assessors, they perform data filtering and processing of state information to facilitate a decision process. The `RouteFlow` function performs assessment functions, however it also performs supervisory functions: it makes decisions on operational flows imposed at flow control structures. A schematic flowchart of the `RouteFlow` function is presented in figure 6.

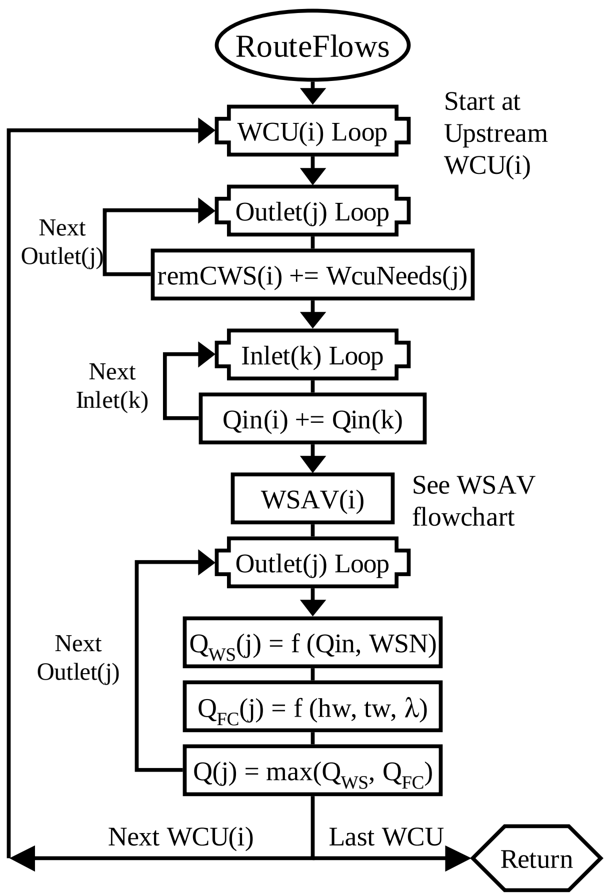

Figure 6: Flowchart of MSE Assessor function: RouteFlows()

`RouteFlow` has a main (outer) loop which processes all WCU's starting at the most upstream point, and sequentially progressing to the most downstream WCU as defined in the MSE Network definition file [SFWMD3 2005].

Since there is no recursive processing involved, `RouteFlow` is not capable of balancing needs and flows that change as a result of the flow decisions made in the single-pass linear processing of WCU's. One of the ways in which this issue is currently addressed is through the use of HSE-MSE iterations described in section 6.

The following descriptions are with respect to a single WCU. The first assessment in `RouteFlow` computes the remaining cumulative water supply needs ($remCWS_i$) of the current WCU based on WCU outlets that are designated as being Water Supply control structures. The computation for WCU with index i that has water supply outlets with index j is:

$$remCWS_i = \sum_{j}^{\mathrm{M}} \beta_j \, CWS_j \tag{11}$$

Once the cumulative downstream needs have been compiled, the water supply needs (WSN) for the WCU is computed by adding the local WCU WSN to the $remCWS$:

$$CWS_i = WSN_L + remCWS_i \tag{12}$$

where WSN_L represents the local WSN computed in `Assess()`.

The next assessment to accumulate the total inflow from WCU inlet structures, these individual values were computed previously for the WCUs immediately upstream of the current WCU:

$$Q_{in_i} = \sum_{j}^{\mathrm{N}} Q_k \tag{13}$$

Followed by a conditional assessment of the water supply available volume ($WSAV$) based on cumulative structural inflows. Q_{in_i} is converted to a water supply available volume ($WSAV_i$) as follows:

$$WSAV_i = Q_{in_i} \Delta t \tag{14}$$

A schematic flowchart of $WSAV_i$ computation is shown in figure 7. Figure 7 shows how $WSAV_i$ is decremented as portions of it are assigned to the current WCU and to downstream WCUs. These assignments depend on the purpose of the current WCU and the magnitude of WSAV compared to needs.

If the current WCU has a maintenance level but no reserve level, then the first priority is to meet needs in the current WCU. Any remaining portion of $WSAV_i$ will be available to meet needs in downstream WCUs. If the current WCU has a maintenance level and a reserve level, and there is not enough water available to meet all needs (i.e. $WSAV_i < CWS_i$), then only a portion of this WCU's needs are met. The portion corresponds to at least the reserve level volume if available. The remaining portion of $WSAV_i$ is available for downstream WCUs. If the current WCU has local excess, it is added to the available volume from upstream (i.e. a negative value is subtracted from $WSAV_i$). For flow-through WCUs, all of $WSAV_i$ will be available to meet needs in downstream WCUs.

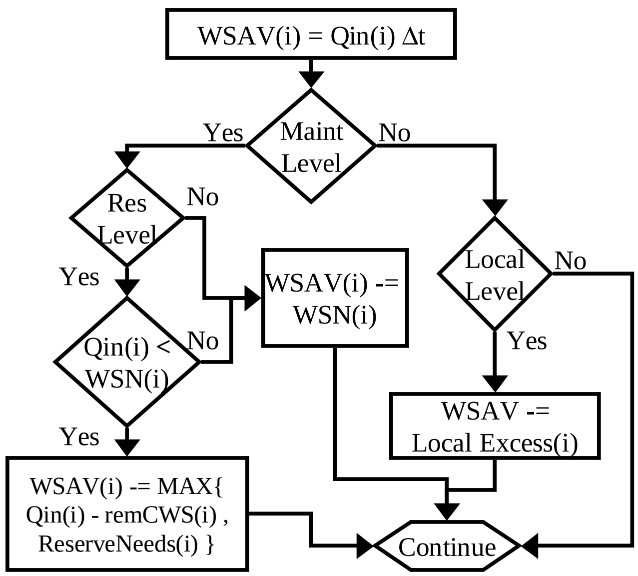

Figure 7: Flowchart of Water Supply Available Volume ($WSAV$) in Route-Flows() function.

Accordingly, $WSAV$ can be expressed as:

$$WSAV_i = \begin{cases} Q_{ini}\Delta\text{t} \ ; & \text{initialvalue} \\ \\ WSAV_i - \text{MIN}(WSAV_i, \\ \text{MAX}[Q_{ini}\Delta\text{t} - remCWS_i, ResNeeds_i]) \ ; & \text{ResLevel} \\ \\ WSAV_i - WSN_i \ ; & \text{MaintLevel} \\ \\ WSAV_i - LEX_i \ ; & \text{LocalLevel} \end{cases} \tag{15}$$

where LEX represents the Local Excess (negative) volume computed by `LocalExcess()`.

4.4.1 WS Flow Computation

At this point in the `RouteFlows()` function the accumulated assessments are completed, the processing now shifts to a supervisory mode wherein the outlet flows are computed for each outlet of the respective WCU's.

The water supply (WS) flow for each water supply outlet structure is based on the available volume of water in the WCU that can be used to meet the cumulative downstream water supply needs. For each WS outlet of the WCU, the available volume (AV) is computed according to a shared adversity assumption:

$$AV_j = (WSAV_j/remCWS) \cdot \beta_j \, CWS_j \quad ; \quad \text{remCWS} > 0 \tag{16}$$

where CWS_j is the cumulative water supply need downstream of the outlet.

The outlet WS flow for the structure with index j is specified as:

$$Q_{WS\,j} = MIN[Q(hw, tw), AV_j/\Delta\text{t}] \tag{17}$$

where $Q(hw, tw)$ is the current state flow capacity reported by the HSE watermover. Note that hw and tw are the latest state estimates from the previous HSE solution, these values may be from sub-timestep iterations as described in section 6. The value of Q_{WS} is then limited by the design capacity of the structure.

The final step of the WS computation is to decrement the $remCWS$:

$$remCWS_i -= CWS_j \cdot \beta_j \tag{18}$$

and to decrement the $WSAV$ of the WCU:

$$WSAV_i -= Q_{WS\,j} \cdot \Delta\mathrm{t} \tag{19}$$

4.4.2 FC Flow Computation

Flood control (FC) flows are based on water levels with respect to the flood control criteria specified for each WCU outlet structure. The criteria are expressed as an open and close water level. The FC flow is:

$$Q_{FC\,j} = \gamma_j \cdot Q_j(hw, tw) \tag{20}$$

where γ represents a fractional value of total flow. γ is based on a fractional gate opening for the structure $fracGO$:

$$fracGO = \frac{hw - \mathrm{close}}{\mathrm{open} - \mathrm{close}} \tag{21}$$

γ is computed as a power function [SFWMD4 1999]

$$\gamma = fracGO^{\mathrm{b}} \tag{22}$$

where b is a parameter usually set to 2. The resultant value of Q_{FC} is limited by the design flow capacity of the outlet structure.

4.4.3 Flow Assignment

Once estimates for the WCU outlet water supply and flood control flows are available, the final outlet flow value is simply a maximum of the two values:

$$Q_j = \mathrm{MAX}[Q_{WS\,j}, Q_{FC\,j}] \tag{23}$$

This value is imposed as a boundary condition on the HSE solution of equation 5 for the structure watermover between the respective WCU canal segments.

5 HSE MSE State Information Mismatch

A distinguishing feature of the HSE is the fully integrated aquifer-stream flow solution. The finite volume hydrological formulation expressed in equation 4 is solved in one step (equation 5) for all waterbodies in the model, inclusive of canal segments and aquifer cells. The HSE solution is therefore an integrated global solution of the simulation hydrologic processes. This feature is desirable from a physical modeling perspective, as the physical system reacts as a global, unitary, fully coupled system.

However, it is problematic from the point of view of MSE which computes watermover flows independently of the conjunctive HSE solution. The essential difficulty is that the MSE decisions are based on previous HSE solution state information, but are imposed on the next HSE solution as flow boundary conditions. As the simulation timestep duration increases ($\Delta \mathrm{t} = \mathrm{t_e} - \mathrm{t_s}$ of equation 2 becomes large) the divergence between the actual cumulative flow $\mathrm{Q_{mn}}$, and the flow estimated by the MSE Assessor Q_{mn} increases. This divergence arises for several reasons:

1. Q_{mn} is based on previous timestep state information.
2. Nonlinearities approximated in the global HSE solution of the equations (equation 9) are not modeled in the MSE Assessor.
3. Lack of synoptic (multiple WCU) balancing of headwater and tailwater in the MSE Assessor.

An observational result of this divergence is the *WCU Profile Mismatch* which precipitates canal stage 'oscillations'. This is described in sections 5.1 and 5.2.

5.1 WCU Profile Mismatch

Consider a coupled set of upstream-downstream WCU's with a single controlled flow structure between WCU's as illustrated in figure 8.

Each WCU has MSE controlled structural inflow and outflow, for example structure $\mathrm{S_{01}}$ controls flow Q_{01} into WCU1, and structure $\mathrm{S_{12}}$ controls flow Q_{12} out of WCU1 into WCU2. These cumulative structural flows are estimated by the MSE Assessor as described in section 4. Each WCU is also subjected to boundary condition and hydrologic state influenced inflows and outflows (Q_s) which include aquifer-canal seepage, rainfall, and all other non-structural fluxes. Operational criteria for a WCU can include a water supply maintenance level $\mathrm{T_{WS}}$, and flood control level $\mathrm{T_{FC}}$ specified at the downstream end of a WCU.

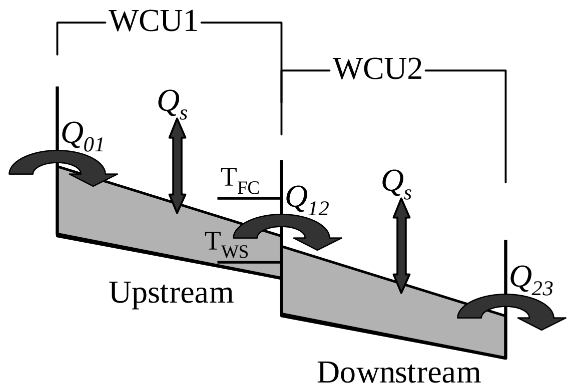

Figure 8: Two WCU's coupled by a structure with flow Q_{12}.

As described in sections 3 and 5, interbasin flow estimates for the next timestep ($Q_{12}(\mathrm{n}+1)$) are based on previous timestep information ($s(\mathrm{n}), Q_s(\mathrm{n})$) and are likely to differ from the actual flows $\mathrm{Q}_{12}(\mathrm{n}+1)$. Further, these estimates are *imposed* as flow boundary conditions on equation 5 for the next timestep (n+1) solution. The imposition of an erroneous estimate can produce significant impacts on the global hydrologic solution. In the case of a positive residual $\Delta Q_{12} = \mathrm{Q}_{12} - Q_{12} > 0$; where the estimated flow is less than the ideal flow, the upstream WCU will contain excess water and will result in a WCU1 water level profile that is higher than that 'expected' by the ideal solution. The deficit of transfer flow from WCU1 to WCU2 will also result in a lower water level in WCU2 than would occur with the correct flow value. This situation is depicted in figure 9.

As a result of the positive flow residual, the headwater of structure S_{12} is above that of the correct value while the tailwater is below the expected value. This increased head differential will produce a larger potential structure flow (the flow produced by application of the structure watermover transfer function to the applied headwater and tailwater) than the correct value. These erroneous water levels, and the incorrect potential flow will be used in the next timestep. Another issue concerns the flood control target level of WCU1. The estimated WCU water level exceeds the flood control level, while the correct value does not. The result would be an incorrect flood control flow release for structure S_{12}.

Consider now a negative flow residual: $\Delta Q_{12} = \mathrm{Q}_{12} - Q_{12} < 0$; where the estimated flow is greater than the actual flow, the resultant WCU water

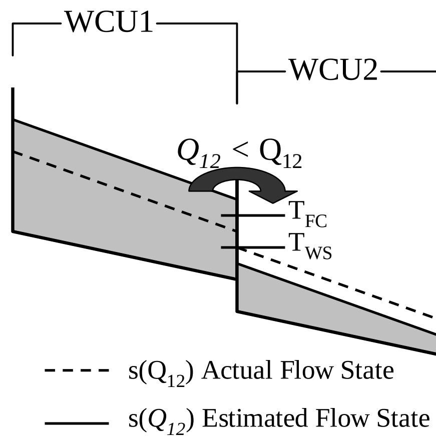

Figure 9: WCU state if estimated flow Q_{12} is less than actual Q_{12}.

levels could be as shown in figure 10.

The negative flow residual has created a water level inversion, the tailwater of structure S_{12} is above the headwater. The potential flow of the structure will be zero. The resultant water level in WCU1 can also fall below the water supply threshold, whereas the actual value would not. The erroneous potential flow and threshold crossing will result in incorrect flow computations on the next timestep.

5.2 MSE Induced Oscillation

There are many causes of canal water level oscillations in hydraulic numerical models, for example, improper spatiotemporal discretizations, numerical threshold crossings, or uncompensated control signals applied to hydraulic flows.

Consider the situation presented in figure 9 where the MSE Assessor encounters a water level in WCU1 above the flood control threshold T_{FC}. The reason for this water level, whether through estimation inaccuracy or levels match the actual values, is not germane. The MSE Assessor will compute a flood control release flow according to equation 20. This value of $Q_{12} = Q_{FC}$ will be relatively large with respect to the structure flow capacity. The large flow can result in significant reduction of water level in WCU1 for the next timestep, lowering the level below the flood control threshold analogous with figure 10. After the next timestep the WCU1 level

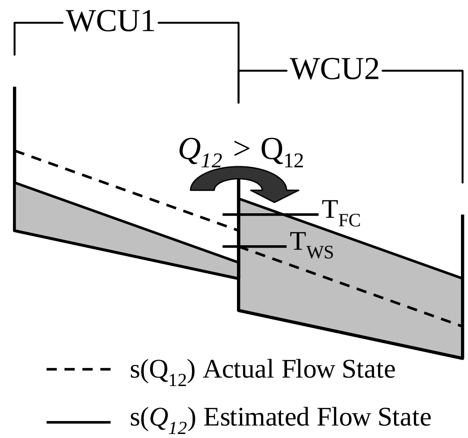

Figure 10: WCU state if estimated flow Q_{12} is greater than actual Q_{12}

may be low enough that no flow release is warranted, thereby setting $Q_{12} = 0$. If the upstream structural and other inflows $(Q_{12} + Q_s)$ are significant enough, then in the following timestep the levels in WCU1 may again rise above T_{FC}, and the cycle repeats.

This cyclic recurrence of control limit states, $Q_{12} = Q_{FC}; Q_{12} = 0$, is commonly referred to as 'bang-bang' control, or 'slamming'. The controller is slamming between maximal control points due to saturation of the control input state variables. Typical solutions entail filtering of the input states, and/or incorporation of an integration term in the control algorithm. In the current MSE Assessor implementation, an alternative approach is used where a convergence function is applied to limit the changes in estimated interbasin flow (Q_{mn}) so that a global solution can be found based on HSE state information feedback (section 6.1).

Even though slamming appears to the primary cause of observed canal oscillations, and this limit cycle behavior is certainly dependent on the nonzero flow estimate residuals, it is possible that the inaccuracies and nonzero flow estimate residuals themselves can produce oscillations as described in section 5.1.

6 MSE - HSE Iterations

Recognizing that the MSE Assessor estimated flow residuals diverge as the simulation timestep increases, a natural solution is to provide iterative HSE state information updates in in order to refine the estimated flows within a timestep. An error metric which quantifies the flow estimate divergence is used to terminate the iterations when a convergence threshold is satisfied. RSM performs this iterative flow refinement in three basic steps:

1. MSE Assessor estimates flows Q_A based on the latest HSE iteration state information (Σ(j-1)) and management constraints (λ).
2. Estimated flow (Q_A) changes are limited with a convergence function to produce the final estimated flow Q.
3. New HSE state estimates are solved by imposition of the estimated flows Q applied to previous timestep state conditions (Σ(i)).

A schematic flowchart of the MSE - HSE iteration is shown in figure 11. Referring to figure 11, there are two processing loops shown. The outer loop represents a HSE timestep and is indexed with the variable i. As i changes from i to i+1, the HSE simulation has advanced forward by one timestep (Δt). The inner loop depicted in figure 11 is the MSE - HSE iteration loop, it is represented with the iteration index j.

The first computation estimates the desired flow Q_A based on the latest state information which will satisfy the operational constraints, thus:

$$Q_A = m[\Sigma(\mathrm{j}-1), \lambda] \tag{24}$$

where $m[]$ indicates the MSE Assessor processing described in section 4.1. The argument Σ(j-1) refers to the HSE state information obtained from the previous (latest) HSE iteration, and as before λ refers to the managerial constraints such as WCU target levels for water supply and flood control. Therefore, the first step consists of estimating the flows required to meet the operational constraints where the state information is taken from the HSE solution based on the most recent MSE Assessor flow estimates. It is assumed that the most recent HSE state information allows better quantification of WCU inflows and outflows than could easily be obtained from the beginning of timestep state Σ(i).

Experience with simulation of Q_A has shown that oscillations of assessed flow (and the corresponding water levels) owing to control point saturation and state variable inaccuracies (see section 5) are common. The second primary computation implements a straightforward, if somewhat inelegant

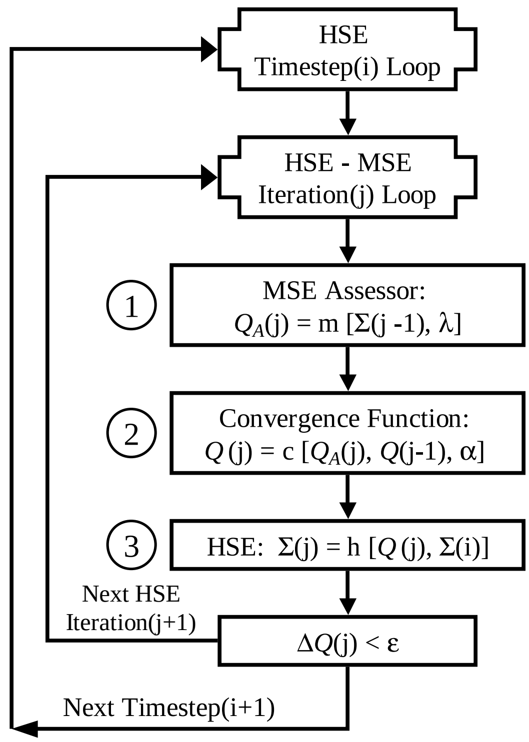

Figure 11: Schematic flowchart of HSE-MSE iteration algorithm.

solution to this problem by imposition of a Markovian weighting to the estimated flow changes:

$$Q(j) = c[Q_A(\mathrm{j}), Q(\mathrm{j}-1), \alpha(\mathrm{j})] = \alpha(\mathrm{j})Q_A(j) + (1-\alpha(\mathrm{j}))Q(\mathrm{j}-1) \qquad (25)$$

where α is a weighting factor in the domain $\Re \subset [0,1]$. The convergence function $c[]$ provides values of α at each iteration as described in section 6.1. One can view this limiting of flow change as consistent with the significant dissipation inherent in the hydrological dynamical system, a requirement for a stable manifold of the chaotic dynamics (see section 3).

Once the estimated, weighted flows are available, the third step is to impose these flows on the HSE state from the previous timestep to compute a current state estimate $\Sigma(\mathrm{j})$:

$$\Sigma(\mathrm{j}) = h[Q(\mathrm{j}), \Sigma(\mathrm{i})] \qquad (26)$$

where $h[]$ indicates solution of the HSE (equation 5) based on the previous timestep state conditions Σ(i), and the MSE Assessor imposed flows Q(j).

The final step is to decide whether or not the estimated flows and resultant states are satisfactory; whether or not to continue the iterations. This is done by comparing the global flow residuals $\Delta Q = \mathrm{Q} - Q$ to a user defined threshold ϵ, where the desired flow value is the unweighted, assessed flow from the MSE Assessor, in other words $\mathrm{Q} = Q_A$ so that:

$$\Delta Q = Q_A - Q \tag{27}$$

This ensures that the final MSE imposed flows converge to the flows which satisfy the operational constraints included in the computations of the MSE Assessor. By basing the convergence criteria on a flow threshold applied to residuals, the user can control a tradeoff between accuracy of the final estimate and the number of iterations. Another result is that the number of iterations is related to the variability of the state conditions. In effect, one can consider the MSE - HSE Iterations as a state dependent implementation of variable timesteps. For example, in relation to the daily timestep Δt = 1 day = 1440 minutes; a terminal iteration at j=144 would correspond to the same computational overhead as a 1440/144 = 10 minute timestep.

It is important to note that the MSE Assessor makes flow estimates based on state information from the previous iteration. As the flow estimates improve and decrease the flow residuals, the accuracy of the WCU inflows/outflows increases. However, at the end of each iteration, the estimated flows are imposed on the previous timestep state conditions. This ensures that the final flows are consistent with the state evolution from the previous timestep to the next timestep.

6.1 MSE Assessor Convergence Functions

The convergence function plays a critical role in allowing the MSE Assessor to find a global solution for flows which are both consistent with the HSE hydrological states, and satisfies the operational constraints. Essentially, the convergence function expressed in equation 25 limits the change in state of the estimated flows from one iteration to the next. This is consistent with the observed nature of the system dynamics wherein dissipation (damping) is inherent. The 'degree of dissipation' is encapsulated in the function α(j). Several different functions for α(j) were evaluated, and this continues to be an area of active development and testing. Some of the functions are described below.

6.1.1 DELTA_Q_ALPHA

This function is based on the change in estimated flow from the (j-1) to (j) iteration. The difference in flow is weighted by the current uncontrolled flow capacity of the structure (C): $\Delta Q_\alpha = [Q(\mathrm{j}-1) - Q(\mathrm{j})]/C$. The flow change is input to a 'bell-shaped' function with the maximal value centered at zero change. The idea is that when $\Delta Q_\alpha \to 0$, the value of α(j) is maximal, which weights the allowed change in flow more heavily to the current iteration value Q(j). As ΔQ_α increases, changes in flow are limited as reliance on the previous value Q(j-1) is increased.

The $\alpha(j)$ function can be expressed as:

$$\alpha(j) = \frac{1}{1+e^{-\mathrm{c}\Delta Q_\alpha}}\left(1 - \frac{1}{1+e^{-\mathrm{c}\Delta Q_\alpha}}\right) \tag{28}$$

where c is a parameter which controls the width of the peak. This function is the derivative of the sigmoid function:

$$S(x) = \frac{1}{1+e^{-\mathrm{c}x}} \tag{29}$$

A plot of α(j) is shown in figure 12.

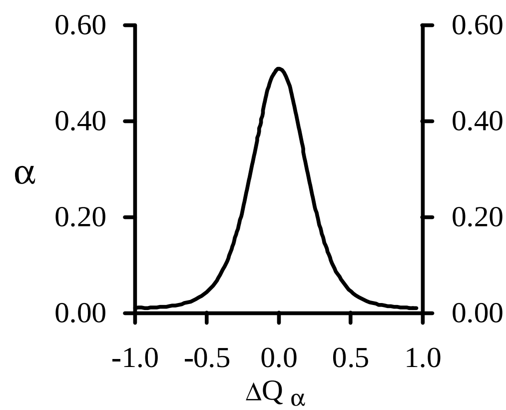

Figure 12: Function α(j) for DELTA_Q_ALPHA convergence function.

6.1.2 INVERTED_SIGMOID_ALPHA

This function computes α(j) based on an inverted sigmoid with an offset iteration $(j - j_O)$ as the sigmoid argument. The idea is to provide a smoothly decreasing function of α as the iterations increase. If the decrease is 'long enough', then the three values Q(j), Q_A(j) and Q(j-1) should converge to the same value. The offset j_O is typically selected to be one half the maximum allowed iterations. The α(j) function can be expressed as:

$$\alpha(\mathrm{j}) = 1 - \frac{1}{1 + e^{-\mathrm{c}(\mathrm{j}-\mathrm{j_O})}} \tag{30}$$

A plot of this α(j) is shown in figure 13 for two values of the parameter c.

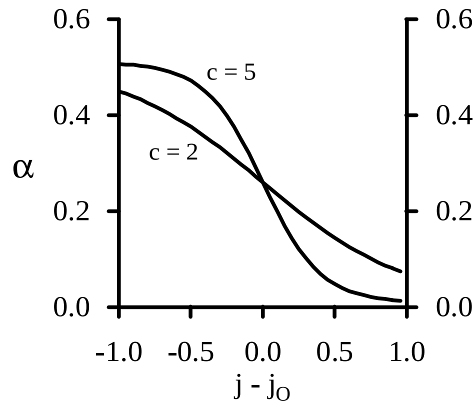

Figure 13: Function α(j) for INVERTED_SIGMOID_ALPHA convergence function.

6.1.3 MOVING_AVG_ALPHA

This function is a generalization of the α(j) function described in section 6.1.2. Instead of centering the function on one half of the maximum allowed iterations, the center is dynamically determined by applying a threshold to a moving average of the flow changes for a particular structure. Each structure maintains a moving average buffer, with the number of points assigned by user XML input, of the change in estimated flow from the previous iteration to the current iteration. When the moving average of this flow change falls below a user specified threshold value, the center of the α(j) function is set to 15 iterations beyond the threshold point.

6.1.4 ALPHA_Q_AVG

This option forms α(j) as an arithmetic average of the previous iteration final estimated flow and the current iteration assessed flow:

$$Q(j) = (Q_A(\mathrm{j}) + Q(\mathrm{j}-1))/2 \tag{31}$$

so that the α(j) function is:

$$\alpha(\mathrm{j}) = 1/2\ \forall \mathrm{j} \tag{32}$$

6.1.5 QDELTA_MAX

This option combines a multi-phase α(j) function with a flow change limiter. The α(j) function resembles a sawtooth as described by:

$$\alpha(j) = \begin{cases} \alpha_0 \frac{\mathrm{N/2-j}}{\mathrm{N/2}}\ ; & \mathrm{j} <= \mathrm{N/2} \\ \alpha_0 \frac{\mathrm{N-j}}{\mathrm{N/2}}\ ; & \mathrm{j} > \mathrm{N/2} \end{cases} \tag{33}$$

where N is the maximum number of iterations and α_0 the maximal value of α(j). The α(j) function is depicted in figure 14.

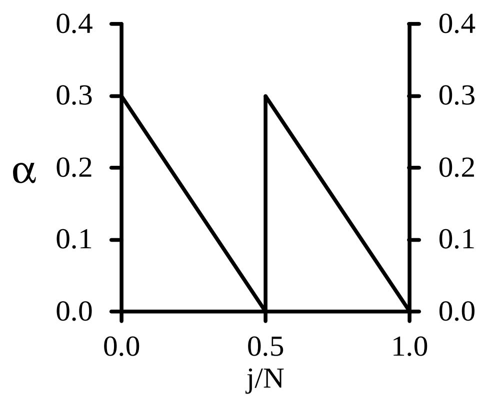

Figure 14: Function α(j) for QDELTA_MAX convergence function.

The first component (sawtooth) is intended to bring the system to a converged state within one half of the maximum iterations. However, if there is not global convergence (all structures in the MSE Network have flow differences between the assessed flows (Q_A) and the imposed value (Q) less than a user specified threshold) then the second sawtooth allows a relaxation of the flow changes to facilitate convergence on the global solution.

Once the α(j) value is available at a particular iteration, the weighted flow estimate is computed according to equation 25. The next step is to compute a flow limit for the structure. The flow limit factor is defined by:

$$\phi(\mathrm{j}) = C_D \left(\frac{\alpha(\mathrm{j})}{100\ \alpha_0} \right) \tag{34}$$

where C_D is the flow design capacity of the structure. Therefore, the flow limit factor ϕ(j) is a percentage of the design capacity weighted by a fraction between [0,1] corresponding to the value of $\alpha(\mathrm{j})/\alpha_0$.

The final step in this method is to apply the flow limiting. The change in flow is limited to $\pm\phi$ if the flow difference between the previous iteration estimate (Q(j-1)) and the current iteration estimate (Q(j)) exceeds ϕ, otherwise, the original estimate from equation 25 is imposed. This computation is expressed below:

$$\alpha(j) = \begin{cases} Q(\mathrm{j}) = Q(\mathrm{j}-1) + \phi(\mathrm{j})\ ; & Q(\mathrm{j}) - Q(\mathrm{j}-1) > \phi(\mathrm{j}) \\ Q(\mathrm{j}) = Q(\mathrm{j}-1) - \phi(\mathrm{j})\ ; & Q(\mathrm{j}) - Q(\mathrm{j}-1) < -\phi(\mathrm{j}) \\ Q(\mathrm{j}) = \alpha(\mathrm{j})Q_A(\mathrm{j}) + \\ \qquad (1-\alpha(\mathrm{j}))Q(\mathrm{j}-1)\ ; & |Q(\mathrm{j}) - Q(j-1)| <= \phi(\mathrm{j}) \end{cases} \tag{35}$$

6.2 Iteration Convergence Example

This section provides visualization of the MSE-HSE Iteration process and the influence of the convergence function. These examples were simulated with the INVERTED_SIGMOID_ALPHA convergence function, applied to the South Dade Conveyance System subregional application. Figure 15 plots the function α(j) for each MSE-HSE iteration.

Figure 16 plots the three estimated quantities Q(j) (green), Q_A(j) (blue), Q(j-1) (red), as a function of the MSE-HSE iteration number for timestep i=282 of the simulation. The total allowed number of iterations was N=144, and the convergence criteria was ϵ=2 cfs. The total number of iterations performed was j=107. The weighted estimates Q(j) and Q(j-1) never deviate very far from their initial, or terminal values, and it is obvious that the threshold deviation between Q(j) and Q_A(j) was achieved before the terminal iteration of j=107. Thus one can expect that this structure was not the one which caused this iteration cycle to require 107 iterations, rather, another

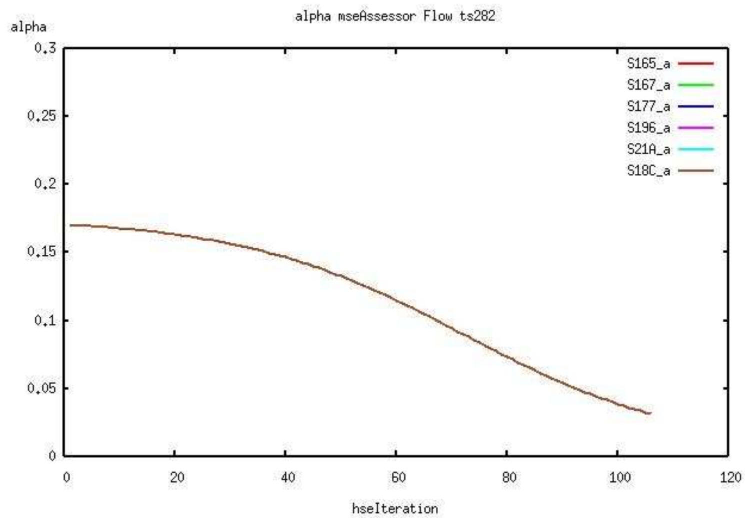

Figure 15: Function α(j) from INVERTED_SIGMOID_ALPHA applied to figures 16 and 17.

structure in the model prevented convergence until j=107. Nonetheless, this provides striking evidence of the difficulties in estimating Q_A as a result of the state information inaccuracies discussed in section 5, the estimates of Q_A are clearly bouncing between the control limits and would perpetuate an MSE induced oscillation without the convergence function. Figure 17 plots another instance of the flow estimates for a high flow structure, S177. The qualitative results are essentially equivalent with those of S166.

6.3 Current Development

Improvements continue to be made as experience is gained in application of the MSE Assessor to subregional models. One of these is decomposition of the convergence function flow threshold into water supply (WS) and flood control (FC) components. Recall that the HSE - MSE iterations have converged when the global flow residuals of equation 27 are below the user defined flow threshold:

$$\Delta Q < \epsilon \tag{36}$$

The number of iterations can be reduced by allowing specification of ϵ_{FC} and ϵ_{WS}, and applying the appropriate threshold to equation 36 depending

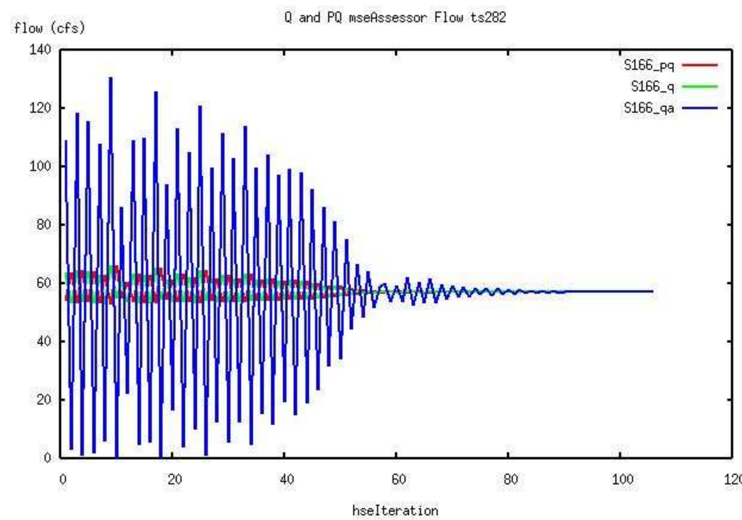

Figure 16: Amplitude of estimated flows Q(j) green, Q_A(j) blue, Q(j-1) red, vs. HSE-MSE iteration for structure S166.

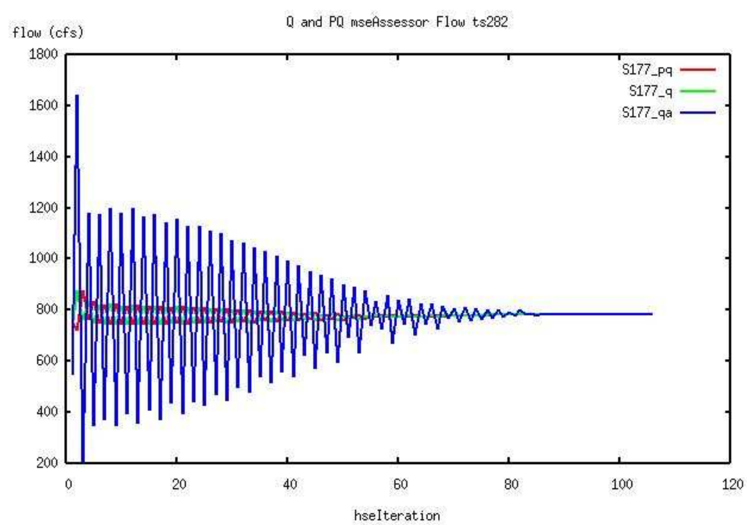

Figure 17: Amplitude of estimated flows Q(j) green, Q_A(j) blue, Q(j-1) red, vs. HSE-MSE iteration for structure S177.

on whether the structure flow is in WS or FC mode.

Another area of active investigation is improvement of the QDELTA_MAX convergence function. Based on results from the South Dade Conveyance System model, the QDELTA_MAX function appears to have the best iteration - convergence and final estimate accuracy.

7 Iteration Reduction via Optimization

8 MathProg Management Expression

8.1 Network Expression

8.2 Objective

8.3 Variables

8.4 Parameters

8.5 Constraints

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