SIMULATION OF OVERLAND AND GROUNDWATER FLOW IN THE EVERGLADES NATIONAL PARK

A. M. Wasantha Lal, Mark Belnap and Randy Van Zee
Lead Engineer, Senior Civil Engineer and Supervising Professional, Hydrologic Systems Modeling Division, South Florida Water Management District.

ABSTRACT

A weighted implicit finite volume model is developed to simulate both overland and groundwater flow in the Everglades National Park (ENP). The model uses triangles to discretize the area, and an external sparse linear equation solver to carry out the final computations. The method is implicit, and is stable under a wide range of triangle sizes, and a wide range of time steps.

The code is written in C++ using object oriented methods. It will eventually be part of a South Florida Regional Simulation Model (SFRSM). The paper includes some early results from the application of the model to the ENP. Results shows that the water levels simulated by the model at a number of selected locations agree with the observed data.

INTRODUCTION

Regional simulation models play a key role in the planning, management and operation of the complex hydrologic system in South Florida. At present, South Florida Water Management Model (SFWMM) is the only available tool for regional analysis of the hydrology of South Florida. The Hydrologic Simulation Engine (HSE) was developed to provide more flexible and detailed analysis within the SFRSM. A number of test cases used to verify the numerical accuracy of the model are presented in the paper by Lal (1998).

A weighted implicit finite volume method is used in the HSE model to solve the diffusion type overland flow and ground water flow equations. The model domain is discretized using triangular cells whose walls control the flow rates into the cells based on the Manning's equation for overland flow and the Darcy's equation for ground water flow. Governing equations are solved simultaneously using an high performance external sparse solver. Results of a preliminary application of the model to the Everglades National Park (ENP) show a good comparison between model results and observed data.

GOVERNING EQUATIONS

St. Venat equations gives the following equation for mass balance of overland flow.

$$\frac{\partial h}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}-S=0 \tag{1}$$

in which u and v are the velocities in the x and y directions; $h=$ water depth; S= summation of source and sink terms including rainfall, infiltration, and evapotranspiration. Once the inertia terms are neglected, Hromadka and Lai (1985), and Lal (1998) showed that the momentum equation can be written in the following form.

$$S_c\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial H}{\partial y}\right)+S \tag{2}$$

in which, K can be computed for overland flow using a general form of the Manning equation $V=\frac{1}{n_b}h^{\gamma}S_f^{\lambda}$ (Kadlec and Knight, 1966) as

$$K = \frac{1}{n_b}h^{\gamma+1}S_n^{\lambda-1} \quad \text{for} \quad \lambda\geq 1 \quad \text{and} \quad |S_n|>\delta_s \tag{3}$$

$$K = K_0 \quad \text{for} \quad \lambda<1 \quad \text{and} \quad |S_n|\leq\delta_s \tag{4}$$

S_n is computed as $\sqrt{(\frac{\partial H}{\partial x})^2+\frac{\partial H}{\partial y})^2}$. $K_0=0$ has been used by Hromadka (1985) and $K_0=h^{\gamma+1}/(n_b\delta_s^{1-\lambda})\;L^2/T$ has been used by Lal (1998) with $\delta_s\approx 10^{-10}-10^{-5}$. $h=0$ for dry cells. For ground water flow, $K=$ transmissivity; $S_c=$ storage coefficient when the water level is below ground; $S_c=1$ at all other times.

THE IMPLICIT FINITE VOLUME METHOD

Using a finite volume formulation, the mass balance condition of 2-D cells describing overland and groundwater flow can be given by

$$\Delta\mathbf{A}\cdot\frac{d\mathbf{H}}{dt}=\mathbf{Q}(\mathbf{H})+\mathbf{S} \tag{5}$$

in which $\mathbf{H}=[H_1,H_2,\ldots H_m\ldots H_{nc}]^T$ is a vector containing the average heads in all the cells; $\Delta\mathbf{A}=$ a diagonal matrix whose element $\Delta\mathbf{A}(m,m)$ is equal to the cell area ΔA_m in the case of a cell m; $\mathbf{Q}$ and $\mathbf{S}$ are the net inflows and source terms to cells. The net inflow rate to a cell m is given by

$$Q_m(H)=\sum_{r=1}^{ns}(\bar{\mathbf{F}}\cdot\mathbf{n})_r\,\Delta l_r \tag{6}$$

$\Delta l_r=$ length of the side r of the triangle; $\mathbf{n}=$ unit outward normal for face r. Cordes and Putti (1996) and Lal (1998) showed that $Q_m(H)$ can be computed using

$$(\hat{\mathbf{F}}\cdot\mathbf{n})_r=\Delta l_rK_r\frac{H_m-H_n}{\Delta d_{mn}} \tag{7}$$

in which Δd_{mn} = distance between circumcenters of triangles m and n; H_m, H_n are the heads at the circumcenters. $K_r = K + K_g$ in which K for overland flow is computed using (3) or (4). K_g = ground water transmissivity. S_n is computed using

$$S_n = \sqrt{\frac{(\hat{H}_j - \hat{H}_k)^2}{\Delta l_r^2} + \frac{(H_m - H_n)^2}{\Delta d_{mn}^2}} \tag{8}$$

In the semi-implicit formulation the computation of flow from cell n to m involves the modification of the following matrix element as it receives water in cell m:

$$M_{m,n} \to M_{m,n} + \frac{K_r \Delta l_r}{\Delta d_{mn}}, \qquad M_{m,m} \to M_{m,m} - \frac{K_r \Delta l_r}{\Delta d_{mn}} \tag{9}$$

Elements $M_{n,m}$, $M_{n,n}$ are modified similarly due to water losses from the donor cell n. Equation (9) can be used with a time weighting factor α to formulate the following system of equations which becomes explicit and implicit for $\alpha = 0$ and 1 (Lal, 1998).

$$[\Delta \mathbf{A} - \alpha \Delta t \mathbf{M}^{n+1}].\Delta \mathbf{H} = \Delta t [\mathbf{M}^n].\mathbf{H}^n + \Delta t (1-\alpha)[\mathbf{M}^n - \mathbf{M}^{n+1}].\mathbf{H}^n + \Delta t [\alpha \mathbf{S}^{n+1} + (1-\alpha)\mathbf{S}^n] \tag{10}$$

Here, $\mathbf{Q}^n = \mathbf{M}^n.\mathbf{H}^n$. The solution $\blacksquare\mathbf{H}$ is used to update the heads using $\mathbf{H}^{n+1} = \mathbf{H}^n + \blacksquare\mathbf{H}$.

USE OF THE OBJECT ORIENTED CODE DESIGN

The C++ language was used in the development of the model, mainly to build flexibility, so that the model could adapt and evolve without drastic modifications. While this flexibility was thought to be necessary after the model was fully developed and throughout its useful life, the advantages of the object-oriented approach have been seen even in the earliest development stages.

The physical domain is ideally suited to be implemented as objects. Different classes describe Elements, Nodes, and Walls. Each of these objects represent a distinct area, location, or boundary, respectively. The interconnections between these objects represent adjacencies in physical, rather than conceptual, location. Each object contains attributes that help it perform a physical based function–Elements are places to store water, Walls are the means of water transfer, and Nodes are location and connectivity placeholders. Figure 1 shows a class diagram and the corresponding representation of an example physical system. The large number of connections between objects allows fast, direct access to information stored in nearby objects.

Because the transfer of water between Elements is through Walls, the Wall class has the responsibility of computing how much flow occurs. To do this it uses a variety of user-selectable methods that model overland and groundwater flow. A linearized coefficient is produced by each of these methods and inserted into $\Delta\mathbf{A}$. The final value stored in $\Delta\mathbf{A}$ is a combination of all the flow methods and indicates the total amount of water transferred between Elements.

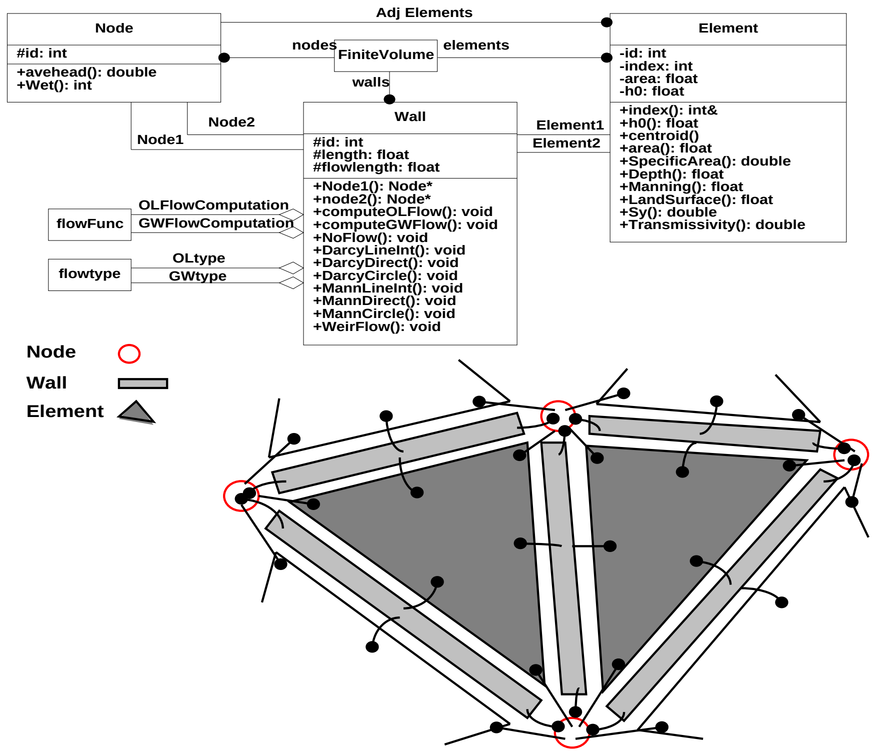

Figure 1: Object diagram and its relationship to the model domain.

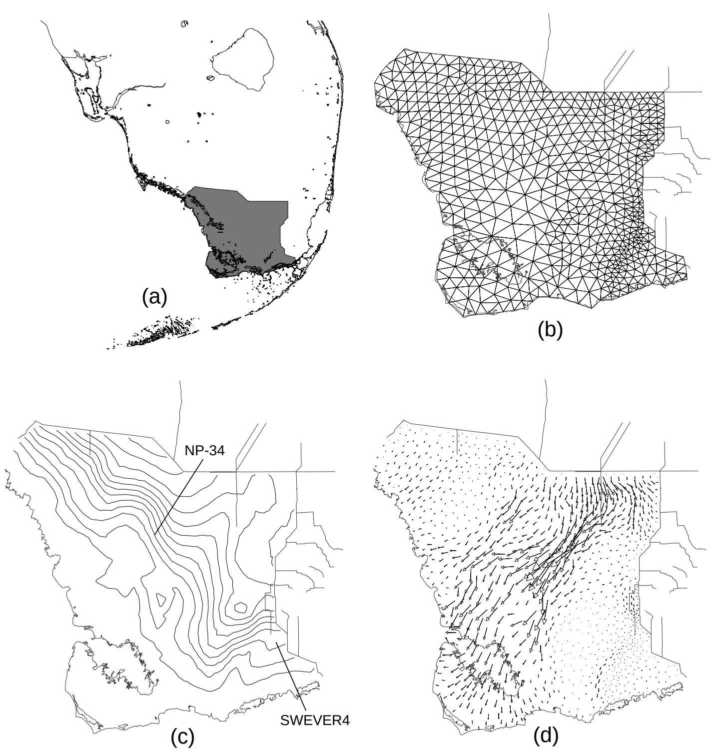

Figure 2: (a) The model location. (b) A finite volume mesh discretization. (c) Land surface elevation; contour interval of 0.25 m. (d) Vector flow field.

APPLICATION TO THE EVERGLADES NATIONAL PARK

Everglades National Park (ENP) is the largest remaining sub-tropical wilderness in the continental United States. This is a flat landscape which was once a 120 mile long, 50 mile wide ”river of grass”. The ENP area shown in Figure 2 is discretized using 1134 triangles, and is subjected to external flow boundary conditions along the Northern and the Eastern boundaries. Boundary inflow/discharges are obtained from the SFWMM simulations. The downstream boundary is assumed to be a free-flow boundary across which water is released to the ocean based on a constant head assumption. A one day time step was used in the simulation for which the daily rainfall and discharge data was also provided at 1 day intervals. The model was calibrated using a conjugate gradient method (Lal, 1995), during which local crop coefficients in the evapotranspiraton equations and the manning’s roughness coefficients were adjusted to reflect local water level variations more accurately in the output.

RESULTS AND DISCUSSION

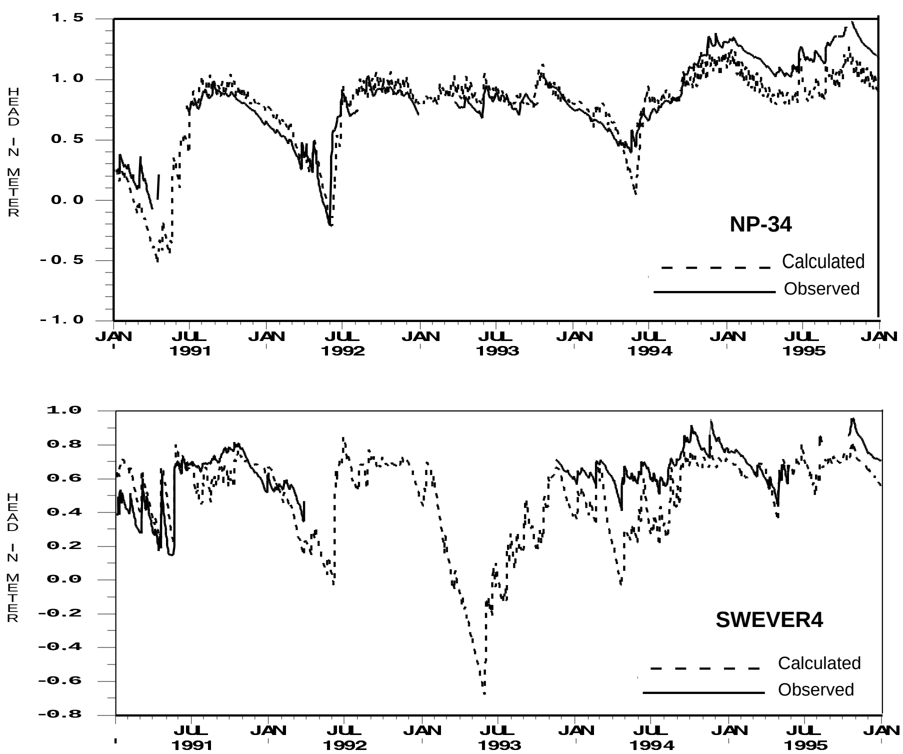

Figure 3: A comparison of results at gages NP-34 and SWEVER-4.

The model output consist of the average water levels in the cells and the water velocities of overland flow and ground water flow components. Figure 2c and 2d show a contour plot of land surface elevations, and a vector plot showing the flow field during a typical wet season day in August, 1991. These figures show detailed flow patterns previously not seen with models that use lower resolutions.

Figure 3a and 3b show a comparison of observed and simulated water levels at two water level at two different regions of the model domain. The figures show that the model output follows the observed values better near center of the undisturbed natural areas than in areas closer to urban influence.

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