# SENSITIVITY AND UNCERTAINTY ANALYSIS OF A REGIONAL SIMULATION MODEL FOR THE NATURAL SYSTEM IN SOUTH FLORIDA

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## ABSTRACT

The sensitivity and uncertainty estimates of a regional model to a selected set of lumped and distributed parameters is determined using the first order method, the Rosenblueth method and the Latin Hypercube sampling (LHS) method. Evapotranspiration (ET) crop coefficients is identified in the study as the parameter most sensitive to the output, followed by roughness. Singular value decomposition is used to determine possible grouping of parameters.

## INTRODUCTION

Parameter uncertainty, combined with input uncertainty and the algorithm uncertainty are responsible for the total model uncertainty. Regional hydrologic models for South Florida use large numbers of distributed parameters such as ET crop coefficients and flow roughness which change with local natural conditions, and cannot be determined with precision. Model outputs also depend on topographic elevations, river widths, detention depths and other features which cannot be determined through direct computations. The study is aimed at determining the sensitivity of selected model outputs to some of the parameters. Model sensitivity analysis provides an opportunity to study the behavior of physical systems to varying natural and man-made conditions.

## METHODS OF ANALYSIS

The simplest method for sensitivity analysis is the first order method in which the following definition of the sensitivity matrix $\mathbf{A} = [a_{i,j}]$ is used.

$$a_{ji} = \frac{\partial y_j}{\partial x_i} \approx \frac{y_j(x_i + \Delta x_i) - y_j}{\Delta x_i} \quad \text{for each} \quad i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m, \quad (1)$$

in which, $y_j, \; j = 1, 2, \ldots, m$ are the $m$ observations; $x_i, \; i = 1, 2, \ldots, n$ are the $n$ parameters. Singular value decomposition of matrix $\mathbf{A}$ can be used to identify groups of parameters sensitive to groups of observations (Lal, 1995). The output variance can be computed approximately using

$$Var(y_i) = \sum_{j=1}^{m} a_{ij}^2 \, \sigma_j^2 + 2 \sum_{j<k}^{m} a_{ij} a_{ik} \, \sigma_j \sigma_k \, \rho_{jk} \qquad i = 1, 2, \ldots n \qquad (2)$$

in which, $\sigma_j^2$ = variance of parameter $j$; $\rho_{j,k}$ = correlation between parameters $j$ and $k$.

The second method used is the factorial design method in which more than one parameter is changed at a time to generate parameter sets for the model runs (Box, et al., 1978). The output is used to determine sensitivity of parameters as well as parameter interactions. The same combinations of parameters are used in the Rosenblueth method to compute output variances (Binley, 1991). Using the method, the $N$ th moment in the output is expressed as

$$E[(y)^N] = \frac{1}{2^m}[(q_{+++\ldots m})(y_{+++\ldots m})^N + (q_{-++\ldots m})(y_{-++\ldots m})^N + (q_{---\ldots m})(y_{---\ldots m})^N] \quad (3)$$

in which, $m$ = number of parameters; $(y_{---\ldots m})$ gives the $N$ th moment when each of the $m$ parameters are set at mean plus or mean minus one standard deviation according to a + and - signs; $q = 1 \pm \rho_{i,j} \ldots$ as described by Binley, (1991).

When using the LHS method (Sing, 1990), probability distributions are assumed for each parameter, and each distribution is divided into a number of intervals. A Gaussian is assumed in the study. Parameters are randomly selected such that each interval appears in exactly one model run. The model outputs generated are statistically analyzed.

## APPLICATION TO SOUTH FLORIDA

Figure 1 shows the area of South Florida studied using the Natural System Model (NSM 4.2). A total of 27 parameters were selected for the study by creating regional parameters for ET and roughness. Overall detention depth and the river width are also considered as parameters. All parameters or groups are varied as percentages of base values. Effect of ground elevation was included in the study by considering unit displacements in several regions. A total of 58 flow, stage and hydro-period observations are used to understand the influence of the parameters.

The first order method is used to estimate the output uncertainty assuming both zero and maximum correlations among parameters to obtain upper and lower bounds of variance. The actual correlations among parameters are not available for the study. When using the Rosenblueth method, parameters of the same type such as ET and roughness were grouped to reduce the 27 independent parameters to $N = 4$ groups, which still requires $2^N = 16$ computer runs. The number of runs made for the LHS method is limited to 200 because each run takes approximately 1 Hr to run in a Sparc 10. Two ranges of uncertainties of input parameters is assumed. The extreme range is based on the assumption that it is physically impossible for the parameters to exceed a certain range. The calibration range is computed considering that any values outside the range would offset the calibration, and become easily noticable. The extreme ranges used for ET, roughness and detention depth are 12%, 43% and 30% of parameter base values. Trimble (1995) used a similar approach, and applied some of the methods to the South Florida Water Management Model (SFWMM).

## RESULTS AND DISCUSSION

The sensitivity matrix for average conditions in Fig. 2 shows the relative importance of ET over other parameters. ET and roughness effects are somewhat spread over the region because many land use types are also spread. Effects of the topographic elevation changes are localized because only small areas were considered. Table 2 shows upper and lower bounds of output uncertainty of selected gages which correspond to zero and maximum correlations. The contributions of individual parameters are also shown in the table. Sensitivity of parameters depend on many state variables too. Water level for example is more sensitive to parameters when it is below the ground than above.

Singular value decomposition of the sensitivity matrix normalized using av-

Table 1: Output uncertainty resulting from assumed parameter uncertainties. Parameters: EFrmN, EFrmS = ET crop coeff. of fresh marsh in the North and the South; ESgrN, ESgrS = ET crop. coeff. of saw grass in the North and the South; MSagrN, MSagrS = Mannings roughness of saw grass in the North and the South.

Observations: QTami, QShSl = discharge across Tamiami Trail and Shark Slough, in ac ft / day; S-10, S3A-28 = stages of gages S-10 and S3A-28 in ft.; T1-7 = hydroperiod at gage 1-7 in days/year.

| | EFrmN | ESgrS | ESgrS | MSagrN | MSagrS | Avg. | $\varepsilon_{uncorr}$ | $\varepsilon_{corr.}$ |
|---|---|---|---|---|---|---|---|---|
| Std. err. | 12% | 12% | 12% | 43% | 43% | | | |
| QTami | 56 | 135 | 173 | 64 | 70 | 3633 | 445 | 1700 |
| QShSl | 23 | 46 | 60 | 23 | 22 | 1300 | 231 | 881 |
| S-10 | 0.08 | 0.02 | 0.00 | 0.03 | 0.00 | 15.0 | 0.68 | 1.04 |
| S3A-28 | 0.01 | 0.03 | 0.04 | 0.02 | 0.00 | 8.4 | 0.13 | 0.48 |
| T1-7 | 15 | 3 | 0 | 3 | 0 | 339 | 18 | 41 |

erages of parameter values and the standard deviations of observations show that the most significant parameter group among the selected consist of almost all the ET coefficients in the central region which remain wet most of the time. This group affects all the stages in the area and the flows. The second parameter group has ground elevation of the central sawgrass region, which mainly affects the local water stages. The singular values for the problem do not decay rapidly as in many small scale models (Lal, 1995) showing that the model actually has a large parameter dimension.

The factorial design method gives more detailed information of sensitivities. For average flow across Tamiami Trail for example,

$$Q_{tam} = 3647 - 7007ET - 784Mann - 56.2Wid + 13.6ET \times Mann + 26.1ET \times Wid + \ldots \quad (4)$$

in which, ET, Mann, Wid = changes of all the values of ET, Manning's roughness and river widths, considered as fractions of base values. The equation shows that interaction terms are relatively negligible, and that sensitivity to one parameter is only slightly affected by another parameter. Equation 4 shows that the average discharge of 3647 Ac ft/day reduces by 7007 Ac ft/day when all the ET values are increased by 100%, assuming a linear model. First order and LHS methods give approximately the same results. The LHS method can also provide estimates of statistical significances and standard deviations.

## CONCLUSIONS

First order methods are simple and fast, but relatively inaccurate. They can provide upper and lower bounds of uncertainty bands when the parameter correlations are not known. Rosenblueth and factorial methods are more accurate, but require a large number or computer runs when the number of parameters is large. Factorial method can detect parameter dependencies as well. LHS method require a large number of computer runs, but this number does not increase exponentially. LHS method gives realistic estimates of parameter uncertainties and significances assuming the parameters to be independent. Since it is not easy to find a single method that can replace all three methods used for sensitivity and uncertainty analysis, it is beneficial to use all three methods at the screening stage, specially when there are many parameters.

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Figure 1: A map of South Florida.
Figure 2: The sensitivity matrix.